Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

16 May 2025

Zone A morning | **Zone B** morning | **Zone C** morning

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Consider the function $f(x)=\dfrac{(x-1)^2}{x}$, where $x \in \mathbb{R}$, $x \neq 0$.

(a) Show that $\dfrac{(x-1)^2}{x}=x-2+\dfrac{1}{x}$. [2]

(b) Hence, find $\int f(x)\,\mathrm{d}x$. [3]

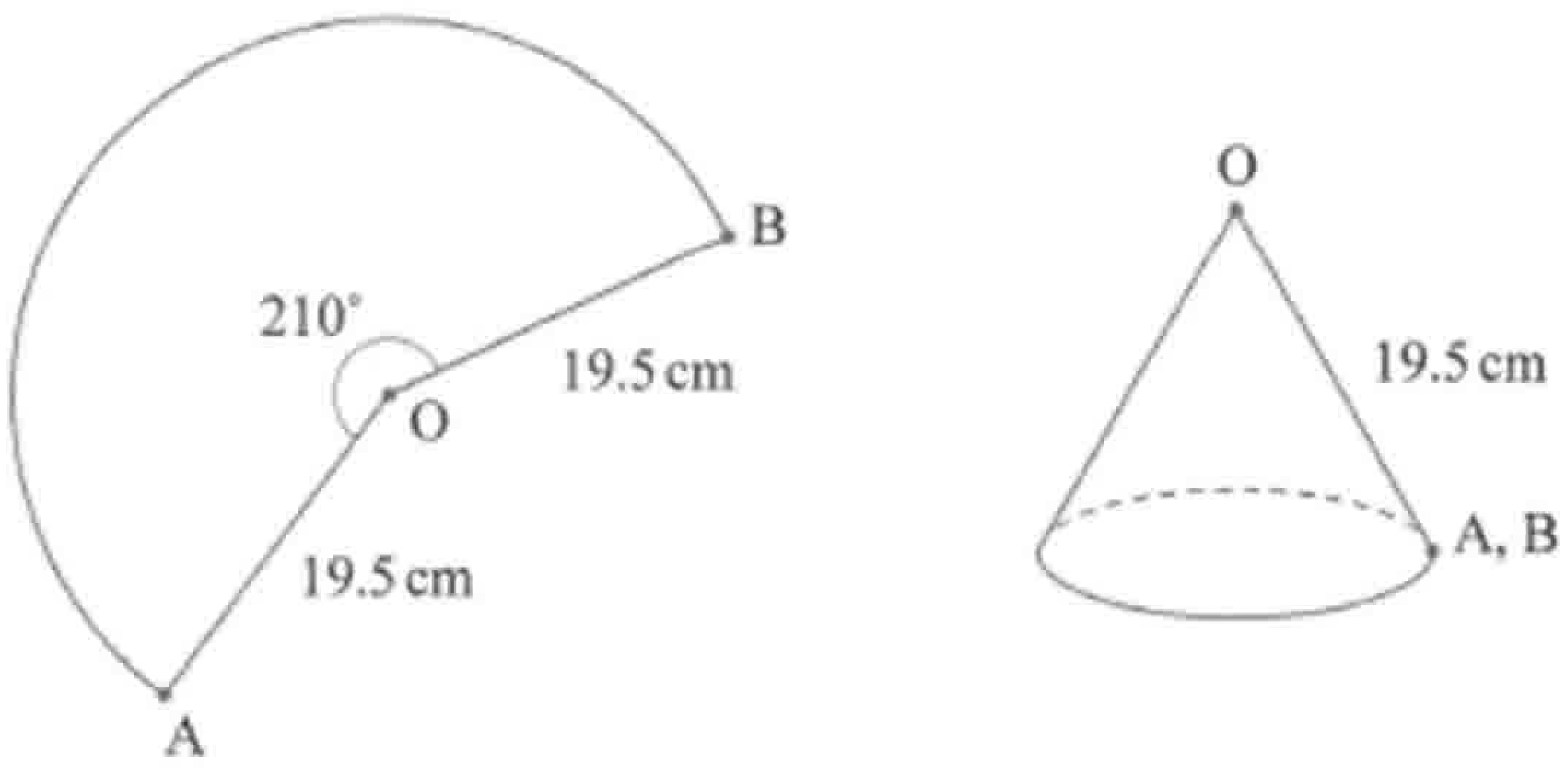
2. [Maximum mark: 6]

The points A and B lie on a circle, with centre O and radius $19.5\,\text{cm}$, such that $B\hat{O}A = 210°$.

A piece of paper is cut into the shape of the sector BOA.

A hollow cone with no base is constructed from the sector by joining the points A and B.
The sector forms the curved surface of the cone.

This is shown in the following diagrams.



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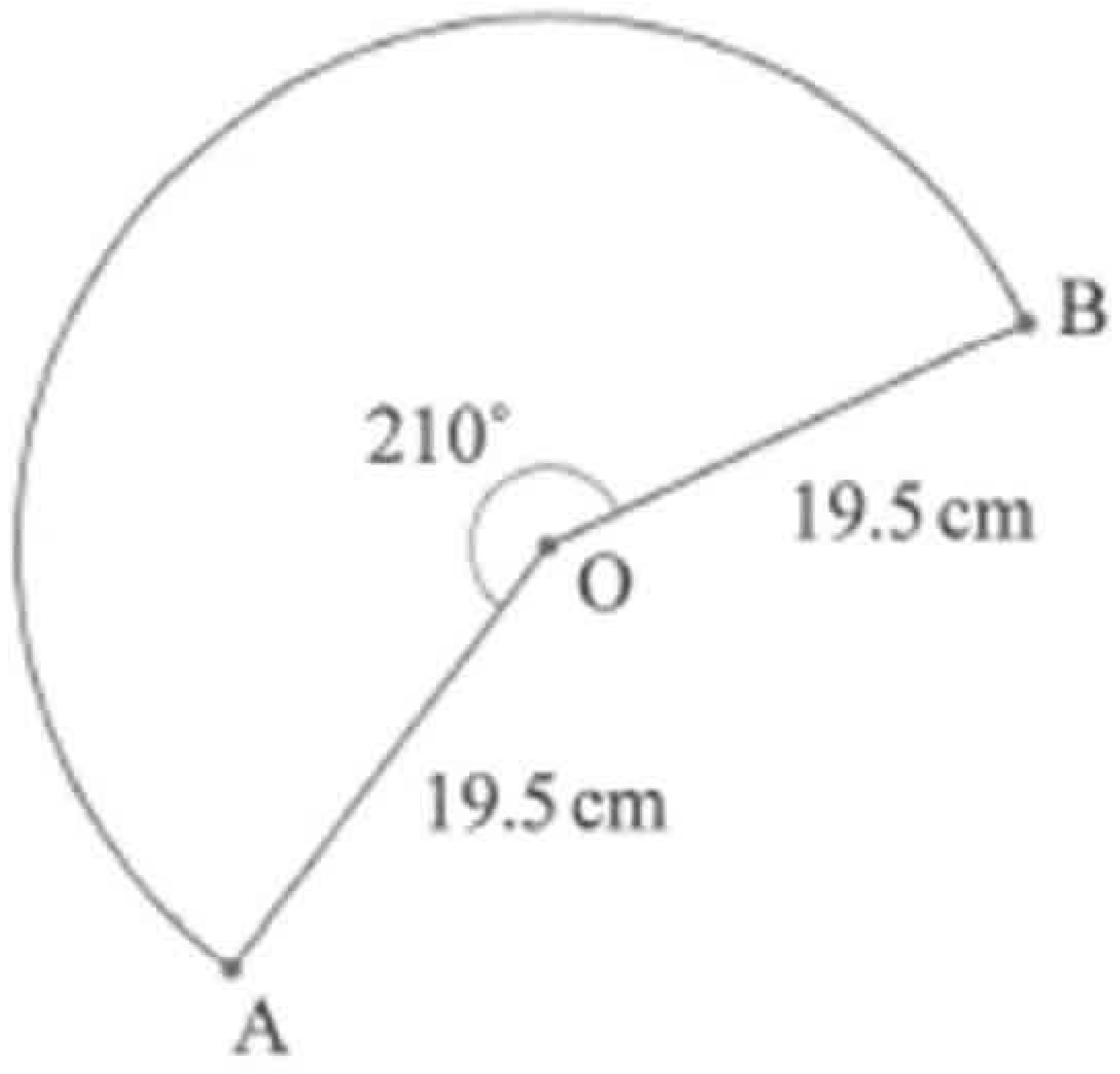

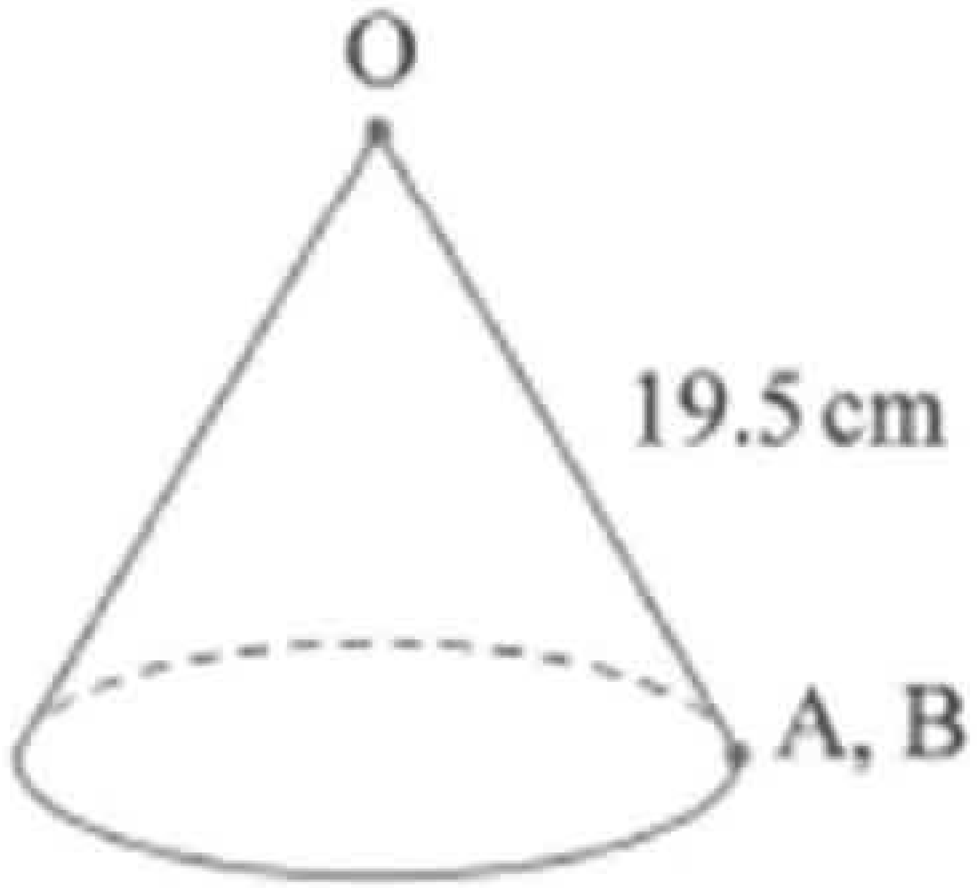

Find

(a) the area of the sector BOA; [3]

(b) the radius of the cone. [3]

3. [Maximum mark: 4]

Consider the function $f(x) = a\tan(2x) + b$, where $x \neq \frac{(2n+1)\pi}{4}$, $n \in \mathbb{Z}$ and $a, b \in \mathbb{R}$.

(a) Write down the period of f. [1]

The graph of $y = f(x)$ passes through the points $\left(\frac{\pi}{12}, 5\right)$ and $\left(\frac{\pi}{3}, 7\right)$.

(b) Find the value of a and the value of b. [3]

4. [Maximum mark: 6]

A population, P, has a rate of change modelled by $\frac{dP}{dt} = -104000e^{-0.0145t}$, where t is the time measured in years since the **start** of 2022.

At the start of 2022, the population was 6.78×10^6.

Based on this model, find the predicted population at the start of 2026.

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5. [Maximum mark: 8]

In a study, measurements for arm span, A cm, and foot length, F cm, are taken from a large group of adults.

For this group, the regression line of F on A is found to be $F = 0.335A - 32.6$, and the regression line of A on F is found to be $A = 2.89F + 99.3$. Each regression line passes through the mean point.

(a) By using an appropriate regression line, find an estimate of the arm span for an adult with a foot length of 19.8 cm. [2]

(b) For this group of adults, find the mean arm span and the mean foot length. [3]

The heights, H cm, of adults in the group can be modelled by a normal distribution with mean 163 cm and standard deviation σ cm.

It is found that 88% of the group have a height between 153 cm and 173 cm.

(c) Find the value of σ. [3]

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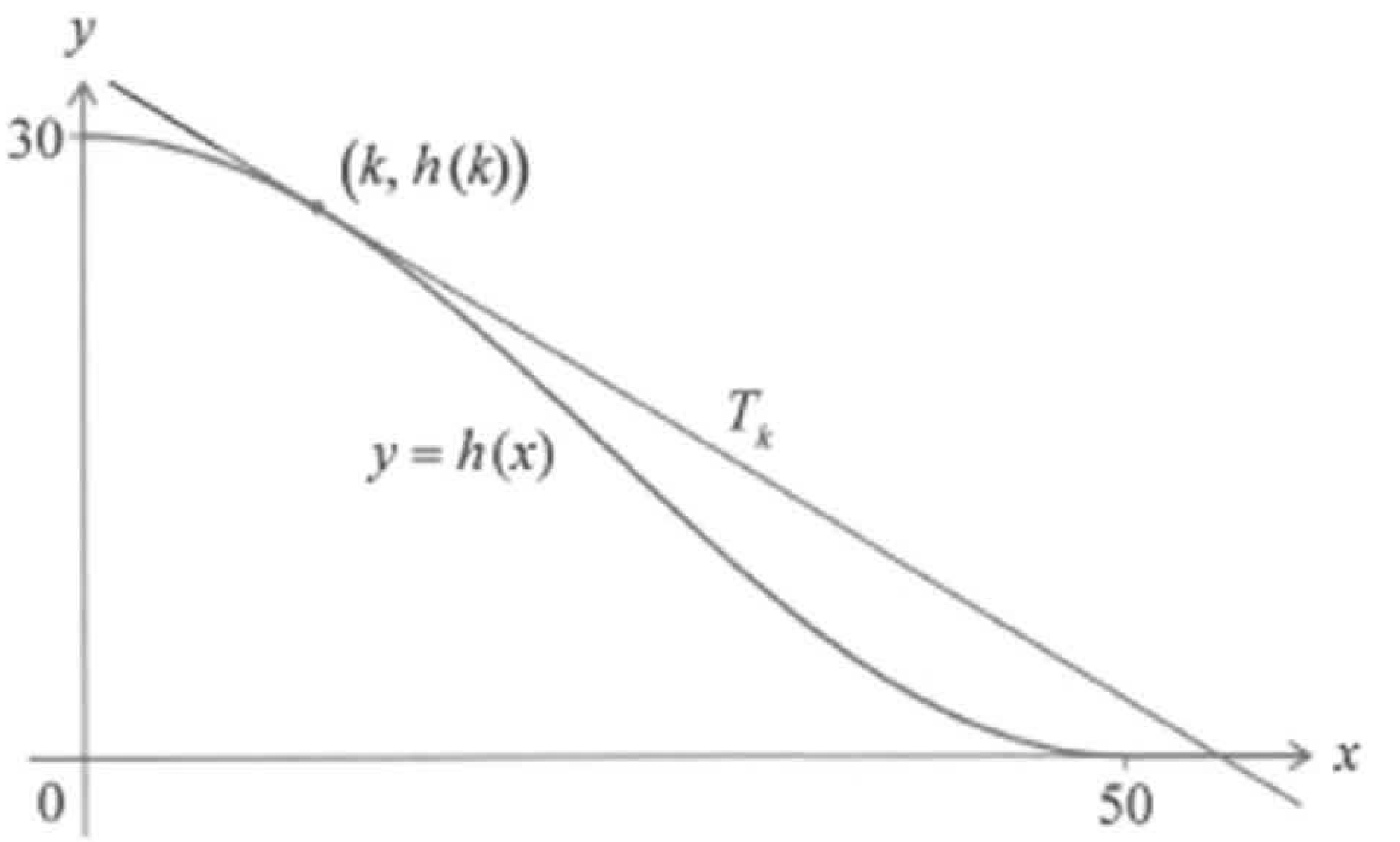
6. [Maximum mark: 6]

Consider the function $h(x) = 15\cos\left(\frac{\pi x}{50}\right) + 15$, where $0 \le x \le 50$.

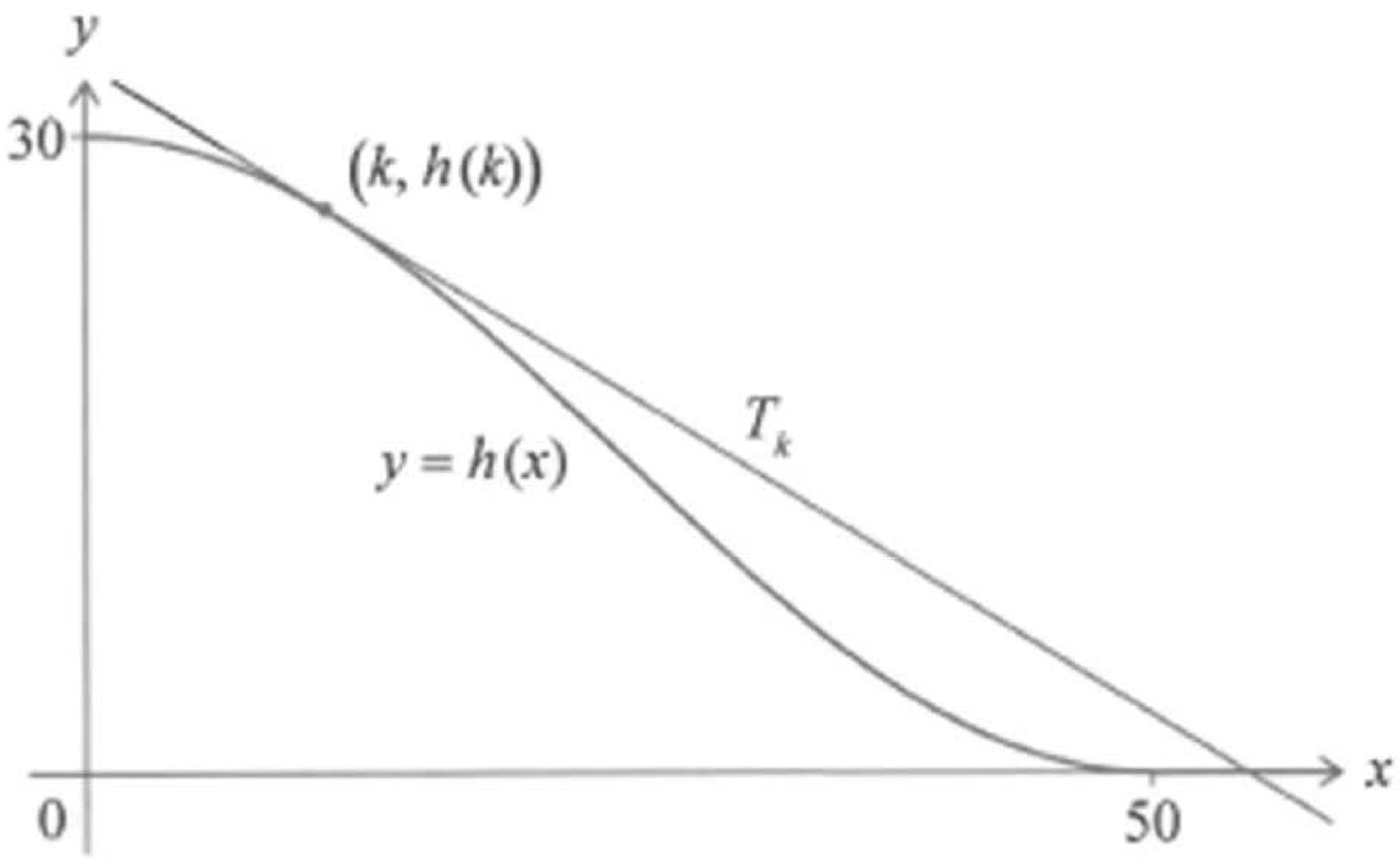
The tangent, T_k, to the curve $y = h(x)$ at the point $(k, h(k))$ is shown on the following diagram.



(a) Find the gradient of T_k in terms of k. [3]

Consider the case where the angle between T_k and the x-axis is $\frac{\pi}{8}$ radians.

The tangent, T_k, to the curve $y = h(x)$ at the point $(k, h(k))$ is shown on the following diagram.



(a) Find the gradient of T_k in terms of k. [3]

Consider the case where the angle between T_k and the x-axis is $\frac{\pi}{8}$ radians.

(b) Find the possible values of k. [3]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

Lynn is playing a game with two unbiased six-sided dice, each with faces marked with the integers from 1 to 6.

In each round, she throws both dice once. The outcomes can be displayed in the following sample space diagram, which has been partially completed:

Die 2

Die 1	1	2	3	4	5	6
1						
2		2,2				
3				3,4		
4						
5			5,3			

Die 2

Die 1	1	2	3	4	5	6
1						
2		2,2				
3				3,4		
4						
5			5,3			
6						

Lynn scores points according to the following rules.

- If the two dice show the same score, she scores 10 points.
- If the two dice show scores which have a difference of one, for example the scores 4 and 5 in any order, she scores 5 points.
- Otherwise, she scores 0 points.

(a) Show that the probability that Lynn scores 5 points in one round is $\frac{5}{18}$. [2]

6						

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- If the two dice show the same score, she scores 10 points.
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- Otherwise, she scores 0 points.

(a) Show that the probability that Lynn scores 5 points in one round is $\frac{5}{18}$. [2]

(b) Find the probability that Lynn scores no points in one round. [2]

The random variable X represents the number of points Lynn scores in one round.

(c) Find $\mathrm{E}(X)$. [4]

(d) Hence, estimate the total number of points that Lynn scores if she plays 90 rounds. [2]

A prize is awarded to any player who scores more than 40 points in total.

Lynn plays exactly five rounds.

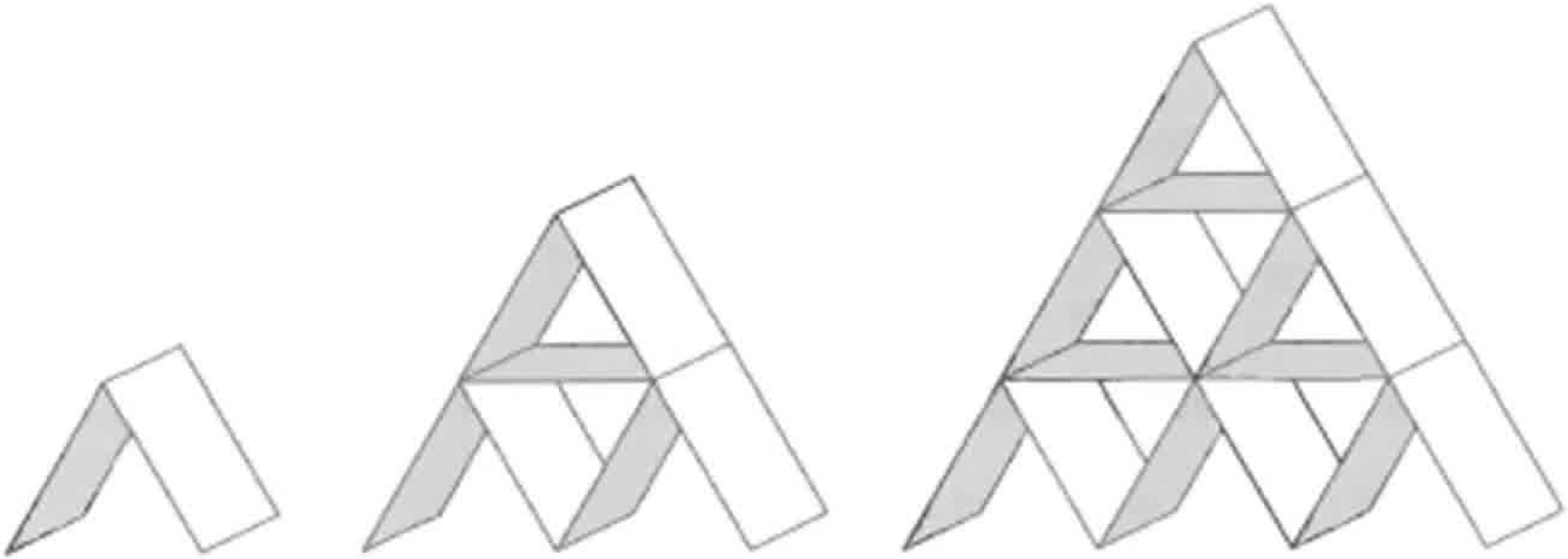
(e) Find the probability that Lynn wins a prize. [4]

8. [Maximum mark: 16]

Rectangular playing cards are stacked in the shape of a pyramid with n rows, where $n \geq 1$.

Some cards are placed horizontally and some cards are stacked at an angle of $60°$ to the horizontal.

The following diagrams represent pyramid stacks for $n = 1$, $n = 2$ and $n = 3$.



$n = 1$ $n = 2$ $n = 3$

Let t_n represent the number of cards used to create a pyramid stack with n rows.

(a) Write down t_3. [1]

(b) Find t_4. [2]

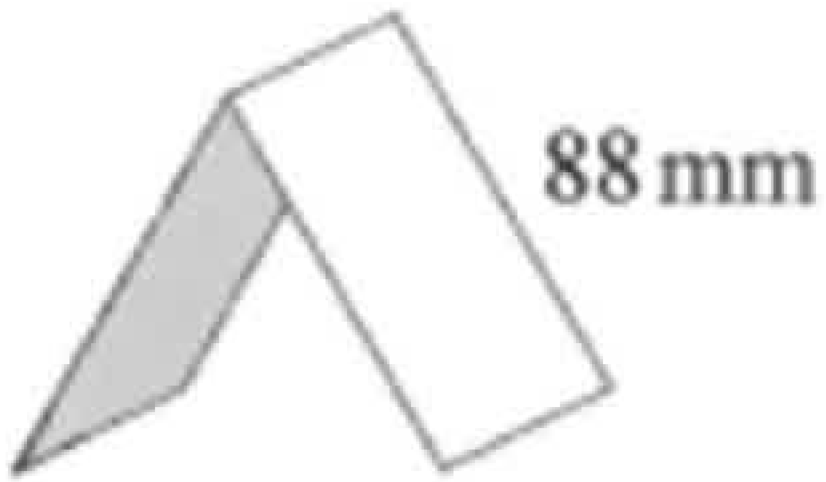
(c) Show that $t_n = \dfrac{n(3n+1)}{2}$. [3]

There are 52 cards in a full pack of playing cards.

(d) A complete pyramid stack is created using playing cards taken from 14 full packs. Find the maximum number of rows in this stack. [3]

(e) A complete pyramid stack is created using playing cards taken from full packs with no cards left over. Find the minimum number of rows in this stack. [2]

The long edge of each playing card measures 88 mm as illustrated in the following diagram.



(f) Find the minimum number of cards needed to create a complete pyramid stack with a vertical height of more than 2 metres. The thickness of the cards may be ignored. [5]

9. [Maximum mark: 15]

Consider the function $f(x) = \frac{2-2x}{x+2}$, where $x \in \mathbb{R}$, $x \neq -2$.

(a) Show that $f^{-1}(x) = f(x)$. [3]

The point $\mathrm{P}\left(k, \frac{2-2k}{k+2}\right)$ is the point on the graph of $y = f(x)$ that is closest to the origin $(0, 0)$.

(b) (i) Find the value of k.

(ii) Hence, write down the coordinates of P. [4]

Consider the function $g(x) = \frac{2-3x}{cx+d}$, where $x \in \mathbb{R}$, $x \neq -\frac{d}{c}$, and $c, d \neq 0$.

The graph of $y = g(x)$ has a vertical asymptote and a horizontal asymptote.

(c) In terms of c and d, write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

It is given that $g^{-1}(x) = g(x)$.

Consider the function $g(x)=\frac{2-3x}{cx+d}$, where $x \in \mathbb{R}$, $x \neq -\frac{d}{c}$, and c, $d \neq 0$.

The graph of $y=g(x)$ has a vertical asymptote and a horizontal asymptote.

(c) In terms of c and d, write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

It is given that $g^{-1}(x)=g(x)$.

(d) Find the value of d. [2]

Consider the case where $c=1$.

(e) Sketch the graph of $y=\frac{1}{g(x)}$, showing the values of any intercepts with the axes and including any asymptotes, labelled with their equations. [4]